5

Optimal Redistributive Tax Transfer Policy

Fellman et al. (1996, 1999) developed an approach to evaluating the design of the tax system and social transfer system scheme by relating the redistributive properties that would have occurred under an optimal design of taxes and transfers. They presented the findings which generalise the findings presented in Fellman (1976) and Fei (1981) and identified the optimal tax and transfer policies in a wide class of possible policies, constrained only to raise a given amount in tax revenue and/or distribute a given amount of cash benefits. These optimal policies for the given budget size would maximize welfare in the distribution of disposable money income in the absence of distinct effects. The extent to which observed policies fall short of this ideal, in reducing inequality, was measured by new indices, in which a distributional judgement parameter can be set to reflect alternative degrees of inequality aversion and to carry out sensitivity analysis.

In the Finnish case for the period 1971-1990, transfers were found not to be very efficient in redistribution income across households, whereas tax policies came much closer to the inequality reducing effect of an optimal pattern (Fellman et al., 1999).

5.1 Optimal Tax Policy

Consider, as above, the before tax income distribution, assumed given with the distribution function $F_X(x)$, density function $f_X(x)$, mean μ_X, Lorenz curve $L_X(p)$ and the Gini coefficient G_X. Following Chapter 3, we consider a class of tax policies characterized by the transformation $Y = u(X)$ where $u(\cdot)$ is non-negative, monotone increasing and continuous with the properties

$$\text{U: } \begin{cases} u(x) \le x \\ u'(x) \le 1 \\ E(u(X)) = \mu_X - \tau \end{cases}, \tag{5.1.1}$$

where $u(x)$ is the post tax income associated with pre-tax income x and the mean tax, assumed given. Consequently, we consider the same class of policies as in (3.1.1). Fellman et al. (1999) considered a slightly different class because the derivative condition was not assumed. Following Fellman et al (1999) we consider here impact effects only, not allowing individual agents for example to adjust their labour supplies in anticipation of the particular tax policy in the class which may be applied.

The polar case presented in (3.1.8):

$$\mathbf{u_0}\text{: } u_0(x) = \begin{cases} x & x \le a_0 \\ & \\ a_0 & x > a_0 \end{cases}, \tag{5.1.2}$$

serves as a reference or benchmark for what follows. Here for incomes $x \le a_0$ there is no tax, but for incomes $x > a_0$ the tax is $x - a_0$. In Section 3.1 we have shown that there exists a unique value a_0 such that $E(u_0(X)) = \mu_X - \tau \le a_0$ (with equality if and only if $F_X(a_0) = 0$), and that the after-tax Lorenz curve given in (3.1.12) is

$$\mathrm{L}_0(p) = \begin{cases} \dfrac{\mu_X}{\mu_X - \tau} L_X(p) & \mathrm{p} \le p_0 \\ & \\ \dfrac{\mu_X}{\mu_X - \tau} L_X(p_0) + \dfrac{a_0}{\mu_X - \tau}(p - p_0) & \mathrm{p} > p_0 \end{cases}, \tag{5.1.3}$$

where $p_0 = F_X(a_0)$. We have shown that the Lorenz curve (5.1.3) is the highest for the whole class of transformations (5.1.1). In Chapter 3 we stated that it

Lorenz dominates the initial income distribution and that irrespectively of the inclusion of the derivative restriction in (5.1.1) or not, the optimal policy is the same. For proofs of these and all subsequent mathematical assertions, see Chapter 3, Fellman (1995) and Fellman et al. (1996).

Although not all members of the class of policies under consideration are progressive, i.e. inequality reducing the policy $u_0(x)$ generates a post-tax income distribution that Lorenz dominates all tax policies of the given class $\boldsymbol{U}$ (Fellman, 1995, 2001; Fellman et al., 1996, 1999). Consequently, it also Lorenz dominate the flat tax policy $\hat{u}(x) = \dfrac{\mu_X - \tau}{\mu_X} x$, whose Lorenz curve is $L_X(p)$. Consequently, $L_0(p) \geq L_X(p)$ and $u_0(x)$ Lorenz dominates the initial income variable *X*.

Following the Atkinson (1970) theorem, (5.1.3) therefore implies maximal social welfare in this class, and $u_0(x)$ is in this sense optimal.

The generalized Gini coefficient of Yitzhaki (1983) for income after this optimal tax policy is

$$G(\nu) = 1 - \nu(1-\nu)\int_0^1 (1-p)^{\nu-2} L_0(p)dp\,, \tag{5.1.4}$$

which may be expressed in terms of the original Lorenz curve $L_X(p)$ using (5.1.3). According to the formula (3.1.21) a lower limit of this generalized Gini coefficient is $G_X(\nu) - \dfrac{\tau(1-G_X(\nu))}{\mu_X - \tau}$. Here ν is a distributional judgement parameter, increases in which connote a more inequality-averse stance on the part of the social-decision maker. The case $\nu = 2$ is that of the ordinary Gini coefficient.

Now consider any actual (non-optimal) tax policy with mean tax τ and let $G_X(\nu)$ and $G_{X-T}(\nu)$ be the generalized Gini coefficients for pre- and post-tax income, respectively. Let furthermore, $G_0(\nu)$ be the generalized Gini coefficient for the optimal policy $u_0(x)$ in (5.1.2). Fellman et al. (1999) proposed to measure the effectiveness of this actual policy by the index:

$$I_T(\nu) = \frac{G_X(\nu) - G_{X-T}(\nu)}{G_X(\nu) - G_0(\nu)} \times 100 \tag{5.1.5}$$

which records its inequality-reducing performance as a percentage $I_T(\nu)$ of the maximum reduction that could have been achieved with the same tax yield τ. This is in contrast with some existing approaches to the measurement of redistributive effect, namely those of Musgrave and Thin (1948), Pechman and Okner (1974) and Blackorby and Donaldson (1984), which express actual inequality reduction as a percentage of pre-tax inequality and equality. In the first two cases cited, the Gini coefficient is used, and in the last the Atkinson index, to measure inequality. See Lambert (2001, Section 8.4) for more of this. Our index thus uses the optimal tax policy as a yardstick, whereas the others use the pre-tax distribution. In fact, the Pechman and Okner construction, if not the other two, use an implicit “optimal” yardstick, in essence comparing actual redistribution with that occurring if all income units were given the same post-tax income (i.e., it uses perfect redistribution, with zero net budget, as a reference). By confining attention to the class of tax policies which satisfy the government budget constraint to assess the effectiveness of an actual tax, or index has, as Fellman et al. (1999) stressed, more realism and direct appeal. It also incorporates the distributional judgement parameter ν which can be varied to carry out sensitivity analysis.

5.2 Optimal Transfer Policy

Consider the income Y with distribution and density functions $F_Y(y)$, $f_Y(y)$, mean μ_Y and Lorenz curve $L_Y(p)$. In conformity with Chapter 4, we study a whole class of transfer policies characterized by a transformation $h(Y)$, where $h(\cdot)$ is non-negative, monotone increasing and continuous with the properties

$$\begin{cases} h(y) \geq y \\ E(h(Y)) = \mu_Y + \rho \end{cases}, \tag{5.2.1}$$

where $h(y)$ is the income, including cash transfer from government, associated with original income y. These properties indicate that no income decreases, that the internal order of the incomes remains the same and that all the policies raise the mean income to $\mu_Y + \rho$, where ρ is the mean benefit, taken as given. The scenario pursued here can apply as well to an income policy: in that case $h(y)$ is the income after a policy-induced increase.

The polar case which serves as a reference or benchmark for what follows is:

$$h_0(y) = \begin{cases} b & y \leq b \\ y & y > b \end{cases} \tag{5.2.2}$$

i.e., all incomes below the level b are raised to b and all incomes above this level remain. It is shown in Section 4.1 that there exist a unique level b_0 such that

$$E(h_0(Y)) = \mu_Y + \rho$$

and for which the Lorenz curve for income including benefits according to (4.1.8):

$$L_0(p) = \begin{cases} \dfrac{b_0}{\mu_Y + \rho} p & p \le q_0 \\ \dfrac{b_0}{\mu_Y + \rho} q_0 + \dfrac{\mu_Y}{\mu_Y + \rho} (L_0(p) - L_0(q_0)) & p > q_0 \end{cases} \qquad (5.2.3)$$

(where $q_0 = F_Y(b_0)$). Fellman et al. (1999) gave a slightly different layout of the Lorenz curve, but the relation $b_0 q_0 - \mu_Y L_0(q_0) = \rho$ between the variables given in Section 4.1 proves the mathematical identity between the proposed formulae. The Lorenz curve $L_0(p)$ is the highest for the whole class of transformations defined by (5.2.1) and higher than $L_Y(p)$, thus engendering highest social welfare in the class (again not all policies in the class (5.2.1) are inequality reducing).

The generalized Gini coefficient for income after this optimal transfer policy is

$$G_0(\nu) = 1 - \nu(1-\nu)\int_0^1 (1-p)^{\nu-2} L_0(p)dp, \qquad (5.2.4)$$

which may be expressed in terms of the original Lorenz curve by using (5.2.3).

As with taxes, the effectiveness of any actual (non-optimal) transfer policy with mean benefit ρ and pre- and post-benefit generalized Gini coefficients $G_Y(\nu)$ and $G_{Y+B}(\nu)$, respectively, may be measured in index form by

$$I_B(\nu) = \frac{G_Y(\nu) - G_{Y+B}(\nu)}{G_Y(\nu) - G_1(\nu)} \times 100. \qquad (5.2.5)$$

Expressing the performance as a percentage of the maximum inequality reduction achievable for a given budget ρ. The index in (5.2.5) can also be used to assess the inequality-reducing performance of an incomes policy $h(Y)$, measured against the optimal income policy $h_0(Y)$ for the same average increase ρ in peoples incomes.

5.3 The Optimal Redistributive Tax-transfer Policy

We characterize each tax and transfer policy by the mean tax τ and the mean transfer ρ where, we assume that $\rho \le \tau$. The transformation of the original incomes can be performed in two steps, first the taxation which reduces mean income from μ_X by an amount τ, and then the distribution of cash benefits so that the mean increases to $\mu_X - \tau + \rho$.

In this situation, the optimal tax and the optimal transfer policy of Sections 5.1 and 5.2 can be joined to given tax and transfer strategy. Under the assumption that both τ and ρ are taken as given, the joint strategy can be proved optimal, and actual combined tax and transfer programs can be gauged against it for their welfare. The rigorous assumption that both τ and ρ are taken as given is necessary for the optimality. Under the weaker assumption that only the difference $\tau - \rho$ is taken as given, perfect redistribution will be attainable (Fellman et al., 1999).

Following Fellman et al. (1999), we start with the taxation. The optimal tax policy Lorenz dominates any other tax policy. Let Y_0 be post-tax income under the optimal tax policy and let Y_u be post-tax income under an arbitrary tax policy. Assume that $E(Y_0) = E(Y_u)$ and denote as above the corresponding Lorenz curves $L_0(p)$ and $L_u(p)$. Under arbitrary taxation the poorest part of the population (after taxes) is poorer than under the optimal taxation (no taxes paid).

If, after taxation, we consider the benefit policy, then for an optimal income distribution, the optimal benefit policy must be performed. This means all benefits must go to the poor. Then the minimum income under the optimal taxation, b_0 (say), is greater than the minimum income under the arbitrary taxation, b_u. Consider the Lorenz curve after the benefit. Let the breaking points in (5.2.3) be q_0 and q_u, respectively. Obviously $q_0 \geq q_u$. Hence, $L_0(p) \geq L_u(p)$ for $p < q_u$ and for $p > q_0$. For $q_u \leq p \leq q_0$ the curved part in $L_u(p)$ is convex and monotone and cannot intersect twice the linear part in $L_0(p)$. Hence, $L_0(p) \geq L_u(p)$ for all $0 \leq p \leq 1$. Consequently, if we join the optimal tax policy and the optimal benefit policy, the joint policy is optimal.

As shown in Fellman et al. (1999) if $b_0 \geq a_0$ then $q_0 = 1$ and $b_0 = \mu_X - \tau + \rho$ in which case the optimal policy creates perfect equality. If, on the other hand, $b_0 < a_0$ then $q_0 \leq p_0$ and the final Lorenz curve L_D is defined by:

$$L_D(p) = \begin{cases} \dfrac{b_0}{\mu_X - \tau + \rho} p & p \leq q_0 \\ \dfrac{b_0}{\mu_X - \tau + \rho} q_0 + \dfrac{\mu_X}{\mu_X - \tau + \rho}\left(L_X(p) - L_X(q_0)\right) & q_0 < p \leq p_0 \\ \dfrac{b_0}{\mu_X - \tau + \rho} q_0 + \dfrac{\mu_X}{\mu_X - \tau + \rho}\left(L_X(p_0) - L_X(q_0)\right) + \dfrac{a_0}{\mu_X - \tau + \rho}(p - p_0) & p > p_0 \end{cases} \quad (5.3.1)$$

They derived, in a short and straightforward manner the result of Fei (1981). The class of combined tax-transfer policies in which (5.3.1) is optimal is Fei ́s class of “equity-oriented fiscal programs”; moreover (5.3.1) is Fei´s “two-valued program" shown to be optimal in his Theorem 7 (whose proof is complex and combinatorial). The analysis of Fellman et al. (1999) thus extends Fei ́s insight to the more general case of fiscal programs with a non-balanced budget, in which the mean excess tax revenue $\tau - \rho > 0$ can be devoted to publicly provided goods and services repayment of debt, etc. Fellman et al. (1999) showed that in the case of these more general fiscal programs, where the tax yield τ and benefit budget ρ are both specified, the two-valued program with "floor value" b_0 and "ceiling value" a_0 (in Fei ́s terminology) is also optimal. In particular, the analysis extends Fei ́s Theorem 4, in which he shows (for $\tau = \rho$) that either $a_0 = b_0$ (the "maximal rational budget" engendering perfect equality) or $a_0 < b_0$. Fei also proves, in his Theorem 5, that a_0 is decreasing and b_0 is increasing, in the common value $\tau = \rho$; our own analysis proves that more generally, a_0 is decreasing in τ and b_0 is increasing in ρ (by construction).

Finally using the generalised Gini coefficient $G_D(\nu)$ for income after the optimal tax and benefit, system namely

$$G_D(\nu) = 1 - \nu(\nu - 1)\int_0^1 (1-p)^{\nu-2} L_D(p)dp \qquad (5.3.2)$$

which is determined by the original distribution $L_X(p)$ according to (5.3.1), the inequality-reducing performance of any actual (non-optimal) combined tax and benefit policy with mean tax τ and mean benefit ρ can be assessed. Let

$$I_{T,B}(\nu) = \frac{G_X(\nu) - G_{X-T+B}(\nu)}{G_X(\nu) - G_D(\nu)} x100 \qquad (5.3.3)$$

be the index, where $G_{X-T+B}(\nu)$ is the generalized Gini coefficient for disposable income after application of the actual tax and benefit policy and $G_D(\nu)$ is the generalized Gini coefficient for the optimal tax and benefit system**.**

In next section we use the index (5.3.3) and present the analysis of the combined effect of taxation and benefit rules in Finland, 1971-1990.

5.4 Empirical Illustration: Finland 1971-1990

Fellman et al. (1999) illustrated their methods using data from Finland from 1971 to 1990. The data used were drawn from the Household Budget Surveys (HBS) in Finland 1971, 1976, 1981, 1986 and 1990, a series of cross-sectional studies which are comparable over time. The income data in these surveys stem from tax and other administrative registers and can be considered to be of high quality. The sample size varies from 1296 in 1971 to 2897 in 1990. The taxation and benefit rules are the rules valid for the period 1971-1990. The sample is restricted to those households with positive disposable income. These data are also used in Example 2.3.1 in Section 2.3.

The base x for taxes includes all taxable income, such as earnings self-employment income, capital income, work-related and taxable transfers and private transfers. From this we subtract direct taxes t to get the base for all non-taxable benefits b. These was taken in this application to be the two major benefit schemes that have remained non-taxable throughout time period covered, namely child allowances and housing subsidies. During the period, 1971-1990, child allowances are paid to the households at a flat rate per each child under the age of 16 (17 in 1990). From the third child onwards, the sum per child increases. Housing subsidies have been means-tested throughout the time period and are therefore negatively correlated with the tax base.

The income variables were standardized to be comparable across households of different sizes using the OECD equivalence scale, which assigns the weight of 1.0, 0.7 and 0.5 equivalent adults to the first and additional adults and children, respectively. Household disposable income per equivalent adult is equal to $x-t+b$ (Fellman et al., 1999).

In Table 5.4.1 we show *inter alia* the effectiveness indices $I_T(\nu)$, $I_B(\nu)$ and $I_{T,B}(\nu)$ estimated by Fellman et al. (1999) from the data (along with some other statistics discussed below). Following Fellman et al. (1999), the threshold for the optimal tax was calculated by the following simple procedure. They fixed the threshold to be equal to the *i*th income unit ́s pre-tax income, $x(i)$ say, and collected all income above $x(i)$ of the income units that have higher income. If the total tax thus collected was higher than the actually collected amount, the threshold was set at $x(i-1)$. This procedure was then repeated until the tax threshold led to less taxes being collected when the threshold was set to $x(k)$. The optimal post-tax income is then $x(i)$ for $i<k$ and $x(k)$ for $i \geq k$. The benefit threshold and post-benefit income distribution were analogously estimated. The effectiveness of the actual tax system measured by our index, i.e., the inequality reduction of actual taxes relative to the optimal policy, declines from 1971 to 1981 and rises thereafter, thus having a slight U-shaped pattern over time. The inequality effectiveness of benefits declined between 1971 and 1990 – with exception of 1981. The combined effectiveness of taxes and transfers followed the same U-shaped pattern as that of taxes alone. For instance, using $\nu=2$, in 1990 taxes achieved a 17.7% reduction in the Gini coefficient on moving from pre-tax to post-tax (but pre-benefit) income relative to the optimal tax policy. On moving from actual post-tax income to post-benefit income is reduced by 4.3% relative to the optimal benefits. On the other hand

moving from pre-tax and pre-transfer income toi disposable income would achieve a 15.2 % reduction in equality relative to the optimal combined tax and transfer policy.

Table 5.4.1 *Redistributive effectiveness of taxes and benefits in Finland, 1971-1990, measured using generalized Gini coefficients.*

		Taxes			**Benefits**			**Taxes and benefits**		
		Actual	**Optimal**	**Maxi-mum**	**Actual**	**Optimal**	**Maxi-mum**	**Actual**	**Optimal**	**Maxi-mum**
ν	Year	$D_T(\nu)$	$I_T(\nu)$	$P_T(\nu)$	$D_B(\nu)$	$I_B(\nu)$	$P_B(\nu)$	$D_{T,B}(\nu)$	$I_{T,B}(\nu)$	$P_{T,B}(\nu)$
	1971	8.8	17.3	0.51	1.7	14.3	0.12	10.4	17.1	0.61
	1976	8.2	12.9	0.63	1.8	13.0	0.14	9.8	13.4	0.73
1.5	1981	7.0	11.2	0.63	3.1	17.9	0.18	9.9	13.1	0.76
	1985	9.9	15.0	0.66	2.6	11.8	0.22	12.2	14.8	0.83
	1990	12.5	17.5	0.71	1.4	8.0	0.17	13.7	16.3	0.84
	1971	7.8	18.3	0.43	1.5	10.5	0.14	9.2	16.8	0.55
	1976	7.8	14.0	0.56	1.5	10.0	0.15	9.3	13.5	0.69
2.0	1981	6.8	12.4	0.55	2.8	13.7	0.21	9.5	13.2	0.72
	1985	9.1	15.5	0.58	2.5	9.7	0.26	11.3	14.3	0.79
	1990	11.5	17.7	0.65	0.9	4.3	0.21	12.3	15.2	0.81
	1971	7.2	19.7	0.37	1.5	9.3	0.16	8.6	16.9	0.51
	1976	7.5	15.1	0.50	1.6	8.8	0.18	9.0	13.8	0.65
2.5	1981	7.1	14.3	0.49	2.6	11.3	0.23	9.5	13.8	0.69
	1985	8.6	16.2	0.53	2.7	9.2	0.29	11.0	14.3	0.77
	1990	11.0	18.6	0.59	0.7	3.1	0.23	11.7	14.9	0.78

Source: Fellman et al. (1999).

Notes. The reduction in equality D is measured as the percentage decline in the generalized Gini coefficient due to actual taxes, benefits, or both. The optimal inequality reduction I is measured as the actual decline in pre-tax, (transfer or tax and transfer) income inequality as a percentage of the optimal decline. See text, especially equations (5.1.5), (5.2.5) and (5.3.3) for exact definitions. The maximal decline P is measured as a proportionate reduction that would occur if the optimal policy were implemented. These are related as $D = I \times P$. Note that D and I are expressed as percentages whilst P is a fraction. Differences between D and $I \times P$ in the reported figures are due to rounding errors.

The inequality effectiveness of benefits is always smaller than that of taxes. This is unsurprising as the actual tax schedule in Finland is progressive during the period covered by the data. However, the main benefit studied, the child allowance, depends only on the number of children in the household. The optimal tax schedule thus only increases, rather than introduces, progressivity, whereas the optimal benefit policy would redistribute child allowances heavily to the lower tail, thus greatly increasing the inequality reduction of the actual benefits. The central argument for this is that the tax rate is related to the individual money incomes and not to the equivalent income calculated for the whole household.

The indices $I_T(\nu)$, $I_B(\nu)$ and $I_{T,B}(\nu)$ presented by Fellman et al. (1999) measure the effectiveness of tax and benefit policies relative to optimal yardsticks which are conditional on the budget sizes ρ and τ. Consider the Pechman and Okner (1974) indices

$$D_T = \frac{G_X - G_{X-T}}{G_X} x100 \quad (5.4.1)$$

of inequality impact (here for taxes). There is a simple relationship between our indices and those of Pechman and Okner (suitably generalized for $\nu \neq 2$). It is as follows

$$D_T(\nu) = I_T(\nu) P_T(\nu) \quad (5.4.2)$$

$$D_B(\nu) = I_B(\nu) P_B(\nu) \quad (5.4.3)$$

$$D_{T,B}(\nu) = I_{T,B}(\nu) P_{T,B}(\nu)\,. \quad (5.4.4)$$

Where the terms

$$P_T(\nu) = \left(G_X(\nu) - G_0(\nu)\right) / G_X(\nu)\,,$$

$$P_B(\nu) = \left(G_X(\nu) - G_1(\nu)\right) / G_X(\nu)$$

and

$$P_{T,B}(\nu) = \left(G_X(\nu) - G_D(\nu)\right) / G_X(\nu),$$

express in proportionate terms the maximum inequality reduction that could have been achieved with the given budget sizes (Fellman et al., 1999).

5.5 Concluding Remarks

Following Fellman et al. (1999) we have demonstrated the properties of optimal tax and benefit policies and shown how to gauge the effectiveness of actual (non-optimal) tax and benefit policies, as well as combined tax-benefit-systems, using the inequality impact of optimal policy as a yardstick. This has resulted in new indices for income taxes which contrast markedly with some existing indices of redistributive effect (progressivity), which either involve no optimal yardstick or at best a very unrealistic one. The new optimal yardstick is, of course, not fully realistic. It serves as a benchmark, just as, for example, the 45° line of perfect equality, though unattainable, is taken routinely as the yardstick against which to measure inequality using the Gini coefficient.

In the case of benefit systems, the indices lend themselves directly to another use: to measure the inequality performance of an incomes policy. Furthermore, all of the indices incorporate an inequality aversion parameter, and can be used to assess the contribution of "targeting" to observed inequality trends, along with that of budget size. Fellman et al. (1999) illustrated this by an application to Finnish data (and showed, incidentally, that the findings were quite robust to changes in the assumed inequality aversion of the evaluator).

Fellman et al. (1999) stressed that all of the constructed indices are impact measures, which take the pre-tax income distribution as exogenous to the choice of tax and benefit policies from classes which would have the given mean budget size (τ or ρ). With more sophisticated modelling, for example of people ́s preferences over consumption and leisure or, more ambitiously, in a computable general equilibrium environment, one could in principle devise indices of policy effectiveness with superior welfare properties – but these would not be measurable from published income data.

Another restrictive assumption of the mathematical modelling is that taxes and government transfers do not disturb the ranking of income units from poorest to richest by their living standards (equivalent incomes). Some lump-sum elements in the tax code (e.g. child allowances) can cause reranking in equivalent income terms, as can benefits going to people on the basis of factors outwith the equivalence scale (e.g. single mothers, the handicapped etc.). By using the Lorenz dominance criterion, Fellman et al. (1999) neglected any wider consideration of social needs.

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